A High Order Conservative Semi-Lagrangian Discontinuous
Galerkin Method for Two-Dimensional Transport Simulations
with Positivity Preserving Property

In this talk, we will present a class of high order conservative semi-
Lagrangian (SL) discontinuous Galerkin (DG) methods for solving multi-
dimensional linear transport equations. The methods rely on a
characteristic Galerkin weak formulation, leading to $L^2$ stable
discretizations for linear problems. Unlike many existing SL methods, the
high order accuracy and mass conservation of the proposed methods are
realized in a non-splitting manner. Thus, the detrimental splitting error,
which is known to significantly contaminate long term transport
simulations, will be not incurred. One key ingredient in the scheme
formulation, borrowed from CSLAM [Lauritzen, Nair & Ullrich, 2010], is the
use of Green's theorem which allows us to convert volume integrals into
a set of line integrals. The resulting line integrals are much easier to
approximate with high order accuracy, hence facilitating the
implementation. Another novel ingredient is the construction of quadratic
curves in approximating sides of upstream cell, leading to quadratic-
curved quadrilateral upstream cells. Formal third order accuracy is
obtained by such a construction. The desired positivity-preserving
property is further attained by incorporating a high order bound-
preserving filter. To assess the performance of the proposed methods,
we test and compare the numerical schemes with a variety of
configurations for solving several benchmark transport problems with
both smooth and nonsmooth solutions. The efficiency and efficacy are
numerically verified.