Covariance Structure Analysis of Climate Model Outputs

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* Presenting poster
Goal
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Simulate computationally efficient projections of future climates.
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Projection (2090) from the Community Earth System Modeling Group (NCAR)
Simulate computationally efficient projections of future climates.

Earth system models running on a supercomputer take several months to output projections of future climates.
Challenges
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Climate model outputs from various ESM groups can show plausible, yet different outcomes of future climates.
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Projections (2090) from the CMIP5 ensemble.
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Projections (2090) from the CMIP5 ensemble.
Statistical Emulator
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Multivariate Normal Sampling (MVN) Scheme

\[ \tilde{y} = \mu + \Sigma^{\frac{1}{2}} \epsilon \]
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\[ \hat{\mu} \quad \text{Multi-model ensemble mean,} \quad \hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} y_i \]
Statistical Emulator

Multivariate Normal Sampling (MVN) Scheme

\[ \tilde{y} = \mu + \Sigma^{\frac{1}{2}} \epsilon \]

\( \hat{\mu} \) \quad \text{Multi-model ensemble mean,} \quad \hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} y_i \)

\( \hat{\Sigma} \) \quad \text{Covariance fitting} \quad \Sigma(\theta), \quad \theta = \{\text{range, sill, nugget}\} \)
Challenges
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Climate model outputs are not independent.
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Model: NorESM1-M

Projections (2090) from the same modeling group (NCC) in the CMIP5-RCP8.5 ensemble.
Challenges

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Projections (2090) from the same modeling group (NCC) in the CMIP5-RCP8.5 ensemble.
Proposed Sampling Scheme
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Weighted multi-model ensemble mean.

\[ \hat{\mu} = \sum_{j=1}^{n} \sum_{k=1}^{m_j} \frac{1}{nm_j} y_{j,k}, \] where n is the number of clusters.
Proposed Sampling Scheme

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Inter-model Comparison
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Compare models using a Riemannian metric between covariance matrices of climate model outputs.
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\[ \Sigma(\theta_i) \in \text{Symmetric Positive Definite Matrices} \]
Inter-model Comparison

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Evaluation: Distance Matrix
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Euclidean distance between climate model outputs
Evaluation: Distance Matrix

Euclidean distance between climate model outputs

Geodesic distance between covariance matrices of climate model outputs
Evaluation: Distance Matrix

Little Contrast

Euclidean distance between climate model outputs

Geodesic distance between covariance matrices of climate model outputs
Evaluation: Distance Matrix

Little Contrast

- More Contrast
- More Dependencies Identified
- Less Bias in the Estimators

Euclidean distance between climate model outputs

Geodesic distance between covariance matrices of climate model outputs
Proposed Sampling Scheme
Proposed Sampling Scheme

Fitting covariance for each climate model output:

Modeling Team - MPI

Manifold of Covariance Matrices

\[ \Sigma - \text{Mean} \]

\[ \Sigma_i \]

\[ \Sigma_j \]

\[ \Sigma_N \]

\[ \Sigma_1 \]

\[ \Sigma_2 \]

\[ \Sigma_3 \]

MRI

CCSM4
Proposed Sampling Scheme

Sample a covariance matrix from a distribution on a manifold.

\[ \hat{\Sigma} \sim \mathcal{N}(\bar{\Sigma}, \Lambda|\Sigma(\theta_1), \ldots, \Sigma(\theta_N)) \]

Fitting covariance for each climate model output:

- Modeling Team - MPI
- CCSM4
- MRI
Evaluation: Semi-variogram Plots
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- Climate output models from the CMIP5-RCP2.6 ensemble
- Samples
- Spread

MVN Sampling

\[ \gamma(h) \text{, Semi-variogram function} \]

Model: GFDL-ESM2G

h, Distance in latitude/longitude units
Evaluation: Semi-variogram Plots

- Climate output models from the CMIP5-RCP2.6 ensemble
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MVN Sampling

Proposed Sampling

\( \gamma(h) \), Semi-variogram function

Model: GFDL-ESM2G

\( h \), Distance in latitude/longitude units
Evaluation: Spatial Field
Evaluation: Spatial Field

Model: GFDL-ESM2G

\( \gamma(h) \)

- Proposed Sampling
- MVN Sampling
Evaluation: Spatial Field

Model: GFDL-ESM2G

- Proposed Sampling
- MVN Sampling

Model: GFDL-ESM2G
Evaluation: Spatial Field

Model: GFDL-ESM2G

Proposed Sampling

MVN Sampling

γ(h) vs. h

Model: GFDL-ESM2G

Proposed Sampling
Evaluation: Spatial Field

Model: GFDL-ESM2G

Proposed Sampling

MVN Sampling

\( \gamma(h) \)
Evaluation: Spatial Field

Model: GFDL-ESM2G

- Proposed Sampling
- MVN Sampling

\( \gamma(h) \)

\( h \)

Graph showing the relationship between \( \gamma(h) \) and \( h \) for both Proposed Sampling and MVN Sampling.
Concluding Remarks
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- Analyzing the covariance matrix of climate model outputs on a manifold can potentially be used to investigate interdependencies in the CMIP5 ensembles.
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• Analyzing the covariance matrix of climate model outputs on a manifold can potentially be used to investigate interdependencies in the CMIP5 ensembles.

• Proposed statistical emulator can potentially mimic the existing ensemble of climate model outputs.
Thank you!
Extra
Intuition Plot

- Sill
- Nugget
- Range
- h