

# NONLINEAR MADDEN-JULIAN OSCILLATION INDICES USING KERNEL METHODS

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**Abstract**—We investigate in this paper the dominant intraseasonal signals in both convection and circulation data using the nonlinear Laplacian spectral analysis (NLSA) method. Three Madden-Julian oscillation (MJO) indices are constructed based on temporal modes extracted from pure cloudiness, lower- and upper-level zonal wind anomalies. All three indices reveal strong intermittency and capture well – through the use of kernel-based similarity methods from machine learning – the inherent nonlinear nature of both convection and circulation in the tropics.

## I. INTRODUCTION

The Madden-Julian oscillation [1] is the dominant winter intraseasonal oscillation (ISO) in the tropics and has a 30-90 day eastward-propagating pattern with zonal wavenumber 1-4. Conventional methods for extracting MJO signals are linear, e.g. empirical orthogonal functions (EOFs), singular spectrum analysis (SSA) [2], however atmosphere-ocean coupled dynamical systems such as tropical convection and circulation are governed by highly nonlinear structures. Since 2004 the real-time multivariate MJO (RMM) index [3] has been the state-of-the-art measure for estimating the strength and phase of the MJO. RMM relies on the leading EOFs – after removal of the annual cycle and interannual variability – of the combined fields of outgoing long-wave radiation (OLR) and zonal winds at 200 and 850hPa averaged over the equator.

## II. DATA AND METHOD

In previous work, in an effort to capture the intraseasonal nonlinear patterns in the tropics, we applied the nonlinear Laplacian spectral analysis [4] to full 2D CLAUS (Cloud Archive User Service) infrared brightness temperature data ( $T_b$ ) over the equatorial band 15°S–15°N without any prior preprocessing, seasonal detrending or latitudinal averaging [5], [6]. In this paper we extend the analysis through NLSA to the circulation

component of the atmosphere and use the same zonal wind fields as in the RMM index, i.e. 200 and 850hPa. We show that NLSA extracts highly intermittent winter ISO signals for each of the three individual components.

For both cloudiness and circulation we use data from January 1, 1984 to June 30, 2006 over the equatorial band 15°S–15°N. Observations are collected at an interval of  $\delta t = 6$  h, producing datasets with  $s = 32,868$  samples over the 23 years. In the tropics, positive (negative)  $T_b$  anomalies are associated with reduced (increased) cloudiness, thus providing a surrogate for tropical convection.

The core of NLSA analysis consists of: 1) time-lagged embedding using the delay method [7], followed by 2) the construction of a set of basis functions using kernel-based techniques from machine learning. While RMM uses the principal components (PCs) of the covariance operator, NLSA employs the Laplace-Beltrami eigenfunctions of a discrete diffusion operator. The eigenfunctions of this operator form a natural orthonormal basis set of functions on the nonlinear manifold sampled by the data, providing superior timescale separation [8]. Such patterns carry low variance and may fail to be captured by variance-based algorithms, yet may play an important dynamical role [9]. A detailed description of the method, i.e. the construction of the kernel similarities and the eigenfunctions  $\phi_i$  of the diffusion operator, is provided in [4]. Along the winter ISO signals, NLSA extracts a hierarchy of eigenmodes at different timescales, ranging from interannual, i.e. ENSO, to diurnal modes. Their analysis is out of the scope of this paper but an extended description can be found in [5].

## III. NLSA-BASED INDICES

The individual indices are constructed from the Laplace-Beltrami eigenfunction pairs associated with the corresponding dominant winter ISO signals:

$$r_t = \sqrt{\phi_1^2(t) + \phi_2^2(t)} \quad (1)$$

for each field:  $r_t^{T_b}$  for infrared brightness temperature,  $r_t^{U^{200}}$  for 200hPa zonal wind and  $r_t^{U^{850}}$  for 850hPa

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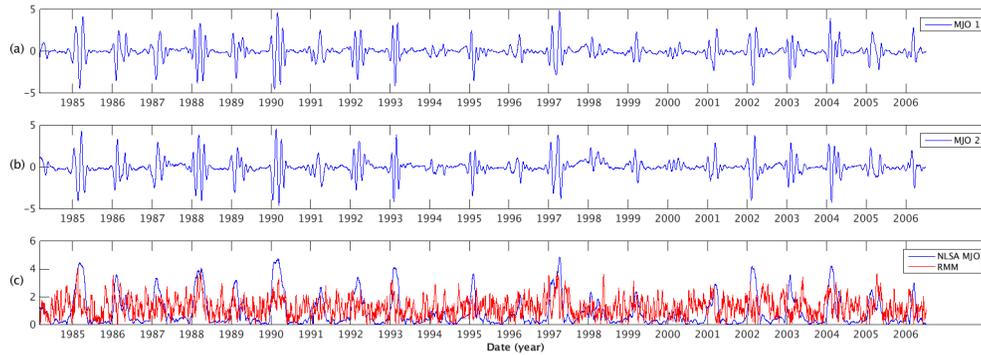


Fig. 1. (a, b) The two NLSA  $T_b$  MJO eigenmodes. (c) The cloudiness MJO index  $r_t^{T_b}$  and the RMM.

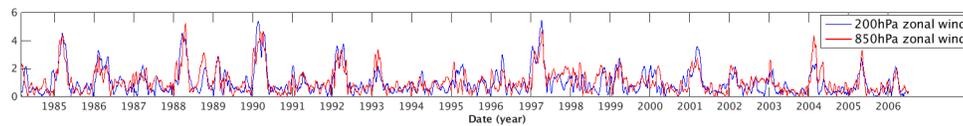


Fig. 2. The 200hPa and 850hPa zonal wind indices,  $r_t^{U200}$  and  $r_t^{U850}$ .

zonal wind. The spatial reconstructions recover the atmospheric circulation associated with the MJO, but the cyclonic/anticyclonic activity around the convection center extends well beyond the equatorial band  $15^\circ\text{S}$ – $15^\circ\text{N}$  into the extratropics.

Figure 1 shows the cloudiness NLSA MJO eigenfunctions and their corresponding index together with the RMM of the combined field. A strong correlation in amplitude between the two indices occurs during boreal winter months, with RMM additionally revealing increased activity during boreal summer. The intermittency is relevant as it reflects naturally the difference between the boreal winter MJO and boreal summer ISO, best reflected in the spatiotemporal reconstructions with an eastward vs. northeastward propagation. The convective pattern that emerges during boreal summer is captured in NLSA by separate eigenmodes [5].

The circulation indices are shown in Fig. 2. The timeseries are slightly noisier compared to the  $T_b$  data however they also display significant intermittency compared to the RMM. Interestingly, the RMM index was documented in [10] to be highly determined by the circulation component, with a 0.99 bivariate correlation between the full RMM index and an RMM index constructed with the OLR field removed. Ongoing work includes combining the convection and circulation NLSA indices into a single index, similar to the RMM, but using nonlinear methods like diffusion maps [11] instead of EOFs. The aim is to compare the combined index with the individual indices presented here and the RMM.

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#### REFERENCES

- [1] R. A. Madden and P. R. Julian, “Detection of a 40–50 day oscillation in the zonal wind in the tropical Pacific,” *J. Atmos. Sci.*, vol. 28, no. 5, 1971.
- [2] M. Ghil *et al.*, “Advanced spectral methods for climatic time series,” *Rev. Geophys.*, vol. 40, 2002.
- [3] M. C. Wheeler and H. H. Hendon, “An all-season real-time multivariate MJO index: Development of an index for monitoring and prediction,” *Mon. Wea. Rev.*, vol. 132, no. 8, pp. 1917–1932, 2004.
- [4] D. Giannakis and A. J. Majda, “Nonlinear Laplacian spectral analysis: Capturing intermittent and low-frequency spatiotemporal patterns in high-dimensional data,” *Stat. Anal. Data Min.*, vol. 6, no. 3, pp. 180–194, 2013.
- [5] E. Szekely, D. Giannakis, and A. J. Majda, “Extraction and predictability of coherent intraseasonal signals in infrared brightness temperature data,” *Climate Dyn.*, 2015.
- [6] E. Szekely, D. Giannakis, and A. J. Majda, “Kernel and information-theoretic methods for the extraction and predictability of organized tropical convection,” *Machine Learning and Data Mining Approaches to Climate Science*, 2015.
- [7] T. Sauer, J. A. Yorke, and M. Casdagli, “Embedology,” *J. Stat. Phys.*, vol. 65, no. 3–4, pp. 579–616, 1991.
- [8] T. Berry, R. Cressman, Z. Greguric Ferencek, and T. Sauer, “Time-scale separation from diffusion-mapped delay coordinates,” *SIAM J. Appl. Dyn. Sys.*, vol. 12, pp. 618–649, 2013.
- [9] N. Aubry, W.-Y. Lian, and E. S. Titi, “Preserving symmetries in the proper orthogonal decomposition,” *SIAM J. Sci. Comput.*, vol. 14, pp. 483–505, 1993.
- [10] K. H. Straub, “MJO initiation in the real-time multivariate MJO index,” *J. Climate*, vol. 26, pp. 1130–1151, 2013.
- [11] R. R. Coifman and S. Lafon, “Diffusion maps,” *Appl. Comput. Harmon. Anal.*, vol. 21, pp. 5–30, 2006.